Inertial confinement in the model of particles interacting at high energy

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Abstract

We consider an application of ideas of inertial confinement from plasma physics for the gas of charged particles interacting at high energy. We formulate this problem as a problem of equilibrium state for the "internal" (slow) particles in some additional potential field created by "external" (fast) particles. In this formulation we find an equation of state for the "internal" particles. We also consider a shock wave description of the process discussing a self-similar solution of the Euler's equations for the fields of interest and we solve Boltzmann equation for self-consistent field (Vlasov's equations) in linear approximation determining an electrical properties of collisionless plasma. Finally we discuss the possibility of the applications of the methods of the approach for the investigation of the early stages evolution of quark-gluon plasma.

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1 Introduction

A collision of heavy nuclei at LHC offers new possibilities to explore a new state of matter, the Quark Gluon Plasma state, which is a dense state of strongly interacting quarks and gluons. Initial stage of scattering of two nuclei is an initial condition for further evolution of the bulk of quarks and gluons in the form of the ideal fluid, see [1]. From this point of view it is interesting to understand a creation of the very dense areas of the mater at initial stages of scattering.

The process of nuclei-nuclei scattering at high energy is a highly non-equilibrium process due to the presence of the partons with different distribution over the particles velocities (rapidities), see [2] for example. In order to describe processes of interactions between these partons the approaches of [3] and [4] can be used. We assume that part of the bulk of the particles form drops of the partons of very high density due to the first initial nuclei fronts interactions. Approximately half of the total initial energy of the interacting protons or nuclei is going there. These particles we can call as internal (”slow”) particles: they loose their velocities during the initial processes of interactions at collision. This picture is due to [4], at initial stage of interactions the ”slow” particles are in the rest in central mass frame, further expansion of these drops is the matter of the hydrodynamical description. There are also partons which we can call as an external or ”fast” ones, they remain relativistic, i.e. they almost do not loose kinetic energy and they do not change their quantum numbers. The influence of these particles in the process of the hot drops compression we can account by their action on the ”slow” partons.

The separation between the internal and external particles, therefore, is based on their sequence in the involvement in the process of interactions and their relative velocities in the laboratory frame. The kinetic energy of the ”slow” particles is heating the volume of the drop whereas the ”fast” particles almost preserve their velocities. Thereby the main difference between these ”slow” and ”fast” particles comes from the value of their relative rapidity after the collision - the secondary ”fast”particles have a large rapidity in relation to the bulk of stopped ”slow” particles immediately after the collision of target and projectile. Now, following to [3], the interactions of the internal particles with external ones we can describe as an equilibrium process for the internal particles in some additional potential field. Namely, the influence of the ”fast” particles we can reduce to some effective potential field included in the description of the stopped matter state. The advantage of this approach is clear, in this way we reduce the non-equilibrium process to the equilibrium one with the possibility to apply a wide variety of approaches knowing for equilibrium description of the matter states. The whole process description, therefore, is similar to the effect of inertial confinement in plasma physics, see [5], where for the creation of the requested density of the matter needed for the thermonuclear fusion a radiation pressure on an external area of a target is used, see the Fig.1.

In our paper, in order to understand a qualitative behavior of the internal particles, we consider an oversimplified model of interacting particle. We assume that the gas of our particles is the gas of

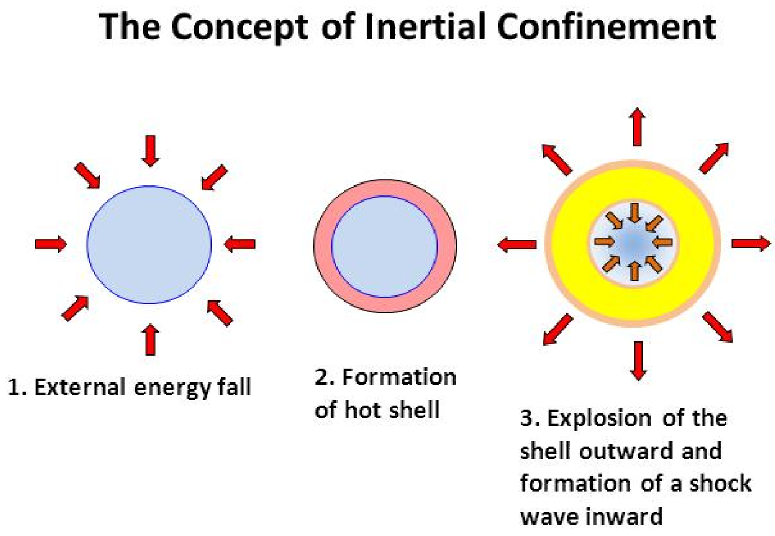

Figure 1: *Inertial confinement in the thermonuclear plasma fusion.*

charged disks with some additional inter-particle interactions which depend on rapidity variable. This rapidity variable characterize the secondary ("fast") particles and related with the total energy of the process. The choose of this variable as the main variable which separate stopped partons from the fast secondaries in the process is dictated by the QCD Pomeron approach in the high energy physics, see [6]. Indeed, the process of interactions between internal and external particles is very similar to the high energy interactions governed by the Pomeron amplitude. In some sense the processes are the same excluding the Pomeron sources terms of the processes. For usual high energy interactions these sources are the target and projectile, whereas in our approximation the sources are the flow of the fast partons and drop of the stopped partons. Nevertheless, in both cases the main variable of the external potential must be a rapidity variable. The value of the potential, therefore, is proportional to the total energy of the process what is also a physically clear and acceptable assumption.

The main aims of our consideration, which can be useful for the better understanding of the properties of quark-gluon plasma, are the understanding of qualitative behavior of matter at initial stage of interactions. In particular we will discuss the transition from gas to liquid description of the interacting matter and some transport properties of the system which can be relevant also at the case of real quark-gluon plasma. The Section 1 is an introduction of the paper. In Section 2 of the manuscript we consider the interactions between our "toy" particles with very general pairwise potential in two dimensional space. The additional external field is chosen to be depended on the rapidity variable and it is the only characteristic of the external particles in the model. For our purposes the form of the potential is unimportant and we chose some simple one. The following subsections of this section are dedicated to the Van der Vaals equation of state of the gas. Section 3 describes thermodynamical properties of the gas. In particular, we discuss transport characteristic of the gas basing on Enskog

approach, see [7]. Section 4 is dedicated to the shock wave description of the process of interactions of external particles with internal ones. Namely, the process of external particles influence on the internal ones is described as a convergence to the center of area of the internal particles shock wave. We consider Euler's equations for the case of this shock wave and discuss the obtained results of this approach following mainly to the results of paper [8]. In Section 5 we generalize the problem and consider Boltzmann equation in Vlasov's formulation for collisonless plasma in three dimensions and describe an equilibrium state of the system[1]. Last section is dedicated to the discussion of the obtained results, there the conclusion of the paper is given.

2 Equation of the state of the gas

2.1 The potential of pairwise interactions

In order to write the equation of state of the gas we need to determine the pairwise potential between two particles, see Appendix A. So, a potential energy of our system we define as a sum of the two parts:

$$U(b) \;=\; \int f(b-x)\, V_1(b)\, f(x)\, d^2\, x \,+\, V_2(b)\,. \tag{1}$$

Here $V_1(b)$ is the usual pairwise potential arising between the particles at the mutual distance b:

$$V_1(b_{12}) \;=\; -A/b_{12} \,+\, B/b_{12}^{\alpha} \tag{2}$$

with $\alpha > 1$ (in our further calculations we take $\alpha = 2$) and A, B are some constants.. The function $f(x)$ is a distribution function of the particles which we take equal

$$f(x) \;=\; \frac{e^{-x^2/R^2}}{\pi\, R^2}\,, \tag{3}$$

see [10] for justification of that choose. The potential energy term $V_2(b)$, absent in Eq. (A.1), describes the potential energy of a single particle inside of some region (with radius R) in the field created by the particles outside the region, i.e. this term accounts an influence of the fast, energetic particles outside of the region with radius R on the particles inside the region[2]. Namely, following the proposition of [3], we reduce a non-equilibrium process of interactions to the equilibrium interactions of only part of the particles in the potential field of external force created by another part of the particles. Following our assumption about the Pomeron like structure of this external field, an additional potential field $V_2(b)$ depends, therefore, only on the position (impact parameter) b of the particle inside of the region and rapidity $Y \,=\, \ln\left(\frac{s}{s_0}\right)$ of outside particles in respect to stopped, "slow", ones, where s as a squared total

[1]In Appendix B, related with the Section 5, we investigate a deviation from the equilibrium state as well

[2]The value of R is somehow arbitrary, it can be considered as radius of proton for example, or as a characteristic value of some hotspot inside the collision region.

energy of the "fast" particles in c.m.f.. This field creates additional pressure on the "inside" particles and could be considered as an analogue of the inertial confinement effect in physics of plasma. For simplicity we choose the form for this term similar to what we have in the phenomenological Pomeron approaches, see [10] :

$$V_2(b) \,=\, -\,\frac{C}{2\,\pi\,R^2}\,e^{Y}\,\theta\,(R^2\,-\,b^2\,) \tag{4}$$

where θ is the step function and C is a constant. In general, the separation to the "internal" and "external" particles depends on a time evolution of the stages of collision, we assume that some region of "slow" particles with high density is produced due very initial stage of nuclei interactions. These particles inside the region we call as "internal", whereas "fast" secondary particles we can call as "external" ones. If, due the interactions, these "fast" particles transfer all the momenta to the particles inside the drop and they also stop then anyway we consider them as external to for the particles inside the initial drop.

For the potential energy we obtain finally the following expression:

$$U(b) = \frac{e^{-b^2/2R^2}}{2\pi\,R^2}V_1(b)\,+\,V_2(b) = \frac{e^{-b^2/2R^2}}{2\pi\,R^2}\left(-A/b + B/b^2\right)\,-\,\frac{C}{2\,\pi\,R^2}\,e^{Y}\,\theta\,(R^2\,-\,b^2\,)\,. \tag{5}$$

Considering an integral Eq. (A.12) with the potential Eq. (5), we see that we can approximate the integral by the following expression

$$J_2\,=\,\int d^2\,b\,(\,\exp\,(-\beta\,U_{12}\,)\,-\,1\,)\,\approx\,-\int_0^{b_0}\,d^2\,b\,-\,\beta\int_{b_0}^{\infty}\,U_{12}\,d^2\,b\,, \tag{6}$$

where b_0 is the value of b where

$$U_{12}(b_0)\,\simeq\,0\,. \tag{7}$$

Thereby, the value of b_0 in Eq. (A.12) we find as roots of the following simple equation:

$$-\,\frac{A}{b}\,+\,\frac{B}{b^2}\,-\,C\,e^{Y}\,\,=\,0 \tag{8}$$

which gives in the limit of large Y (high energy limit):

$$b_0(Y)\,\approx\,e^{-Y/2}\left(\sqrt{\frac{B}{C}}\,-\,\frac{A}{2C}\,e^{-Y/2}\right)\,=\,\tilde{b}_0(Y)\,e^{-Y/2}\,. \tag{9}$$

Clearly, at very large values of final rapidity Y we obtain that $b_0\,\ll\,B/A$, that means $b_0\,\ll\,1$. The reason of this decrease of the inter-particle distance is the presence of the additional pressure introduced in the model.

2.2 Equation of state of the gas

Now, we ready to calculate the integral Eq. (A.12):

$$J_2\,=\,\int d^2\,b\,(\,\exp\,(-\beta\,U_{12}\,)\,-\,1\,)\,\approx\,-\int_0^{b_0}\,d^2\,b\,-\,\beta\int_{b_0}^{\infty}\,U(b)\,d^2\,b\,= \tag{10}$$

$$-\pi\, b_0^2 + \frac{A\beta}{R^2}\int_{b_0}^{\infty} e^{-b^2/2R^2}\, db - \frac{B\beta}{2\,R^2}\int_{b_0}^{\infty} \frac{e^{-b^2/2R^2}}{b^2}\, db^2 + \frac{C\beta\, e^Y}{R^2}\int_{b_0}^{\infty} \theta\,(R^2\, -\, b^2)\, b\, db\,.$$

Performing the integration and keeping in the integrals only leading terms of answer's expansion at large Y ($b_0(Y) \ll 1$) we obtain:

$$J_2 \,\approx\, -\pi\, b_0^2 \,+\, \beta\, \frac{A}{R}\sqrt{\frac{\pi}{2}}\left(1 - \sqrt{\frac{2}{\pi}}\,\frac{\tilde{b}_0}{R}e^{-Y/2}\right) \,+\, \beta\,\frac{B}{2R^2}\left(\gamma\, -\, \frac{\tilde{b}_0^2}{2R^2}e^{-Y}\right) \,+ \tag{11}$$

$$+\,\frac{\beta\, e^Y\, C}{2\,R^2}\,\Big(R^2\, -\, b_0^2(Y)\Big)\,.$$

Using shorter notations we write the answer as

$$J_2 \,\approx\, -\pi\, b_0^2 \,+\, \beta\,\Big(\tilde{A}(Y)\, +\, \tilde{B}(Y)\, +\, \tilde{C}(Y)\, e^Y\Big)\,. \tag{12}$$

Therefore, the state equation of the gas in our case has the following form:

$$P \,=\, \frac{NT}{S}\, -\, \frac{N^2\, T}{S^2}\,\Big(-\pi\, b_0^2(Y)\, +\, \beta\,\Big(\tilde{A}(Y)\, +\, \tilde{B}(Y)\, +\, \tilde{C}(Y)\, e^Y\Big)\Big)\,, \tag{13}$$

or, using the shorter notations again, we have:

$$P \,=\, \frac{NT}{S}\, -\, \frac{N^2\, T}{S^2}\,\Big(-\,\pi\, b_0^2(Y)\, +\, \beta\, a(Y)\Big)\,. \tag{14}$$

Introducing the gas density variable

$$\rho \,=\, \lim_{N,S\to\infty}\frac{N}{S}\,, \tag{15}$$

we rewrite the state equation in the following form:

$$\Big(P\, +\, \rho^2\, a(Y)\Big)\,\Big(1\, -\, \rho\,\pi\, b_0^2(Y)\Big)\, =\, \rho\, T\,. \tag{16}$$

This equation of state of the gas is a modified Van der Vaals equation with coefficients of equation which are depend on the rapidity of the process.

2.3 Critical parameters of the equation of state and thermodynamical parameters of the system

The critical parameters of the equation of the state we find similarly to the usual definition of critical parameters in the Van der Vaals equation

$$-\,\rho^3\, +\, \frac{\rho^2}{\pi\, b_0^2}\, -\, \rho\frac{P_{cr}\,\pi\, b_0^2 + T_{cr}}{a\pi\, b_0^2}\, +\, \frac{P_{cr}}{a\,\pi\, b_0^2}\, =\, (\rho_{cr}\, -\, \rho)^3\,, \tag{17}$$

that gives the well-known answers

$$\rho_{cr}\, =\, \frac{1}{3\pi\, b_0^2(Y)}\,,\quad P_{cr}\, =\, \frac{a(Y)}{27\,\pi\; b_0^2(Y)}\,,\quad T_{cr}\, =\, \frac{8\, a(Y)}{27\,\pi\; b_0^2(Y)}\,. \tag{18}$$

Now, using Eq. (9) and approximating $a(Y)\,\approx\,\tilde{C}\, e^Y$ we obtain

$$\rho_{cr}\, =\, \frac{1}{3\pi\,\tilde{b}_0^2}\, e^Y\,,\quad P_{cr}\, =\, \frac{\tilde{C}}{27\,\pi\;\tilde{b}_0^2}\, e^{5Y/2}\,,\quad T_{cr}\, =\, \frac{8\,\tilde{C}}{27\,\pi\;\tilde{b}_0^2}\, e^{5Y/2}\,. \tag{19}$$

We see that all the values of the critical parameters in the critical point of gas-liquid phase transition are much higher then in the case of usual Van der Vaals equation.

3 Entropy and transport properties of the gas

The state equation of the gas, Eq. (16), we rewrite it in the following form

$$P \,=\, Z(Y,T)\,\rho T\,, \tag{20}$$

where

$$Z(Y,T) \,=\, \frac{1}{1\,-\,\rho\,\pi\,b_0^2(Y)}\,-\,\beta\,\rho\,a(Y) \tag{21}$$

The internal energy (per particle in one mole) of the gas with two transverse degrees of freedom , therefore, is given by usual caloric equation

$$U \,=\, \frac{P\,S}{N\;Z(Y,T)} \,=\, \frac{P}{\rho\,Z(Y,T)} \tag{22}$$

where the Gibbs equation for U takes place

$$dU \,=\, T\,ds\,+\,\frac{P}{\rho^2}d\,\rho \tag{23}$$

with s as an entropy. Using the caloric equation Eq. (22) together with the state equation Eq. (20), we could rewrite the Gibbs equation for the entropy Eq. (23)

$$ds \,=\, \frac{1}{T}\,dU\,-\,\frac{P}{T\,\rho^2}d\,\rho \tag{24}$$

in the following form:

$$ds \,=\, \frac{dP}{P}\,-\,\frac{d\eta}{\eta}\,-\,\frac{1}{Z}\,\frac{dZ}{d\eta}\,d\eta\,-\,Z\,\frac{d\eta}{\eta} \tag{25}$$

where m is a particle mass and $\eta(Y) \,=\, \pi\,b_0^2(Y)\,\rho$ we could call as a packing factor of the problem. Integrating on of the Gibbs relation Eq. (25) gives for the entropy per particle in the gas

$$s \,=\, \ln \frac{P}{\eta\,Z(\eta)\,\exp\left(\int\,\frac{Z(\eta)}{\eta}\,d\eta\right)}\,+\,s_0\,, \tag{26}$$

see [8]. At high energy limit an integrating over η gives for the entropy

$$s \,=\, \ln \frac{P\,\left(1\,-\,\eta\right)^2}{\eta^2}\,+\,s_0\,. \tag{27}$$

At high energy limit, when $\eta \,\ll\, 1$, using Eq. (9) we obtain the following expression for the entropy

$$s \,=\, \ln \frac{P}{P_0}\,+\,2\,Y\,-\,2\,\ln\left(\pi\,\tilde{b}_0^2(Y)\,\rho\right)\,+\,s_0\,, \tag{28}$$

which linearly depends on the total rapidity Y of the process.

The equation of state, Eq. (16) or Eq. (20), we can rewrite in the following form at the limit of high energy:

$$P \,=\, \rho T\,\left(1\,+\,\eta\,Z(\eta)\right)\,, \tag{29}$$

where

$$Z(\eta)\,=\,\frac{1}{1\,-\,\eta}\,. \tag{30}$$

In this equation the compressibility factor $Z(\eta)$ from Eq. (29) coincides with the Enskog factor in Eq. (20). Therefore, the transport properties of the gas, which are characterizing by shear viscosity ζ, bulk viscosity ξ and heat conductivity κ we take in Enskog approximation from [7]:

$$\zeta\,=\,\zeta_0\,\eta\,\left(\frac{1}{\eta\,Z(\eta)}\,+\,1\,+\,0.87\,\eta\,Z(\eta)\right)\,, \tag{31}$$

$$\xi\,=\,\xi_0\,\eta\,\left(\,1.25\,\eta\,Z(\eta)\,\right)\,, \tag{32}$$

$$\kappa\,=\,\kappa_0\,\eta\,\left(\frac{1}{\eta\,Z(\eta)}\,+\,\frac{3}{2}\,+\,0.87\,\eta\,Z(\eta)\right)\,. \tag{33}$$

The Enskog approximation, therefore, allows to find the relative changes of the transport coefficients of the system under the pressure whereas the initial values of these coefficients are given. At the limit of high energy, whereas $\eta\,\ll\,1$, keeping only leading in η terms, we obtain:

$$\zeta\,\approx\,\zeta_0\,, \tag{34}$$

$$\xi\,=\,1.25\,\xi_0\,\eta^2\,Z(\eta)\,\approx\,1.25\,\xi_0\,e^{-2Y}\,\left(\pi\,\tilde{b}_0^2(Y)\,\rho\right)^2\,\left(1\,+\,e^{-Y}\,\pi\,\tilde{b}_0^2(Y)\,\rho\right)\,, \tag{35}$$

$$\kappa\,=\,\kappa_0\,\left(1\,+\,\frac{1}{2}\,\eta\right)\,. \tag{36}$$

We see, that the bulk viscosity ξ is small at the limit of large final rapidity Y in comparison to the case when $Y = 0$ for our gas.

Now we can consider the obtained results for the entropy and transport coefficients properties from the point of view of results for quark-gluon plasma models. In [9] was obtained that $\xi\,/\zeta$ ratio has a maximum at some critical temperatures T_c. Considering the same ratio constructed from Eq. (34)-Eq. (35) values we obtain that this ratio is equal to

$$\frac{\xi}{\zeta}\,=\,\frac{1.25\,\xi_0\,e^{-2Y}\,\left(\pi\,\tilde{b}_0^2(Y)\,\rho\right)^2}{\zeta_0} \tag{37}$$

and that it is small. Moreover, considering the critical parameters of gas-liquid transition Eq. (18) we see, that this ratio has a maximum at $\rho\,=\,\rho_{cr}$ and at critical temperature T_c[3]:

$$\frac{\xi}{\zeta}\,=\,\frac{1.25\,\xi_0}{9\,\zeta_0}\,, \tag{38}$$

which is similarly to the properties of the same ratio in the quark-gluon plasma model of [9]. Thereby our simple model demonstrate properties which are similar to the properties of much more complicated models of quark-gluon plasma. Physically it means, that in Enskog approximation the bulk viscosity

[3]The critical temperatures in [9] and here have different meanings, but both of them define higher temperature comparing to initial one.

is proportional to the density of the state and the maximum values of density and bulk viscosity are achieved at liquid state of the system. Further, when the state of non-compressible liquid is achieved due the evolution, the density and bulk viscosity of the liquid will remain constants.

Additionally it is interesting to note that whereas in our model the entropy is a growing function of the total rapidity of the process, see Eq. (28), the shear viscosity Eq. (34) is a constant. It provides a small $\zeta\,/\,s$ ratio

$$\frac{\zeta}{s} \propto \frac{\zeta_0}{Y} \tag{39}$$

as it found in hydrodynamical description of quark-gluon plasma evolution, see in [1] for example.

4 Shock wave point of view

In this section we proceed following to main results of paper [8]. Our flow of compressed interacting particles could be described by the following Euler's equations for the radial velocity, mass density

$$\partial_t \sigma \,+\, \partial_r(\sigma\, u) \,+\, \frac{1}{r}\,\rho\, u \;=\, 0\,, \tag{40}$$

$$\partial_t\, u \,+\, u\,\partial_r\, u \,+\, \frac{1}{\sigma}\,\partial_r\,\sigma \;=\, 0\,, \tag{41}$$

and, assuming an absence of dissipative processes for our gas, for the entropy

$$\partial_t\, s \,+\, u\,\partial_r\, s \,=\, 0\,. \tag{42}$$

Here u is the radial velocity of the flow, $\sigma \,=\, \rho\, m$ is the mass density of the gas and s is the entropy from Eq. (27):

$$s \,=\, \ln \frac{P}{\sigma^2\, \exp(C(\sigma))} \,+\, s_0 \tag{43}$$

with

$$C(\sigma) \,=\, -\, \ln\Big(1 - \sigma\, \pi\, b_0^2(Y)/m\Big)^2\,. \tag{44}$$

There are three partial differential equations for the main fields which characterize the flow of interacting particles under the external pressure. Solution of this system is considered in the next subsection.

4.1 Shock wave description of the process: self-similar solution

In this section we consider the process of gas compression as the process of propagating convergent radial shock wave, which can be described in term of scaling variable

$$\xi \,=\, \frac{r}{r_{shock}} \,=\, \frac{r}{A\,(t_0 \,-\, t)^{\alpha}} \tag{45}$$

where A and α are some constants to be determined further, an implosion of the shock wave occurs at time $t = t_0$ which corresponds to $\xi = \infty$, and the front of shock wave is described by the radius:

$$r_{shock} = A\,(t_0 - t)^{\alpha}\,. \tag{46}$$

The fields of interests are assumed to have the following form

$$\sigma = \sigma_0\, G(\xi)\,, \tag{47}$$

$$u = \frac{\alpha\, r}{t - t_0}\, V(\xi)\,, \tag{48}$$

$$P = \frac{\alpha^2\, r^2}{2\,(t - t_0)^2}\, \sigma_0\, G(\xi)\, W(\xi)\,, \tag{49}$$

with functions $G(\xi)\, V(\xi)\, W(\xi)$ to be found from the equations Eq. (40) - Eq. (43) which we rewrite in the following form:

$$\frac{dV}{d\ln\xi} + (V - 1)\,\frac{d\ln G}{d\ln\xi} + 2V = 0\,, \tag{50}$$

$$(V-1)\,\frac{dV}{d\ln\xi} + \frac{W}{2}\frac{d\ln G}{d\ln\xi} + \frac{1}{2}\frac{dW}{d\ln\xi} + W - V\left(\frac{1}{\alpha} - V\right) = 0 \tag{51}$$

$$\frac{dW}{d\ln\xi} - \frac{d\ln G}{d\ln\xi} - \frac{dC}{dG}\,\frac{dG}{d\ln\xi} + \frac{2}{\alpha}\,\frac{1 - \alpha V}{1 - V} = 0\,. \tag{52}$$

The system of differential equations Eq. (50) - Eq. (52) can be linearized by expansion of the functions V, W around $V(\xi \to \infty) \to 0$ and $W(\xi \to \infty) \to 0$ that corresponds to $t \to t_0$ time in Eq. (48) - Eq. (49) , see more details in [8]. The solutions of the linearized equations can be easily found, they are

$$V \simeq \frac{K_V}{\xi^{1/\alpha}}\,, \tag{53}$$

$$W \simeq \frac{K_W}{\xi^{2/\alpha}}\,, \tag{54}$$

with constants K_V and K_W must be found from the full equations. Concerning the G function from Eq. (47) we note, that this function remains finite at $\xi \to \infty$:

$$\frac{d\ln G}{d\ln\xi} \to 0 \tag{55}$$

when $V(\xi \to \infty) \to 0$ and $W(\xi \to \infty) \to 0$.

In order to solve the equations Eq. (50) - Eq. (52), we need initial values of our functions $V(\xi = 1), Z(\xi = 1), G(\xi = 1)$ which could be determined from the matching equations on the discontinuities of the flow:

$$\sigma_1\, u_1 = \sigma_2\, u_2\,, \tag{56}$$

$$P_1 + \sigma_1\, u_1^2 = P_2 + \sigma_2\, u_2^2\,, \tag{57}$$

$$U_1 + \frac{P_1}{\rho_1} + \frac{m\, u_1^2}{2} = U_2 + \frac{P_2}{\rho_2} + \frac{m\, u_2^2}{2}\,. \tag{58}$$

with subscript 1 for the quantities before the shock and subscript 2 for the quantities after. The solutions of these equations in the limit $P_2 \gg P_1$ were found in [8] and they are the following:

$$\sigma_2 \,=\, \sigma_1 \left(1 \,+\, \frac{2}{Z(\eta_2)} \right)\,, \tag{59}$$

$$P_2 \,=\, \frac{2\,\sigma_1\,u_1^2}{2\,+\,Z(\eta_2)}\,, \tag{60}$$

$$u_2 \,-\, u_1 \,=\, -\,\frac{2\,u_1}{2\,+\,Z(\eta_2)}\,, \tag{61}$$

$$u_1 \,=\, -\,\dot{r}_{shock} \,=\, \frac{\alpha\,r_{shock}}{t\,-\,t_0}\,. \tag{62}$$

Additional equation which relates the densities of the flow is Eq. (47):

$$\sigma_2 \,=\, \sigma_0\,G(1) \,=\, \sigma_1\,G(1) \tag{63}$$

that together with Eq. (59) gives:

$$G(1) \,=\, 1\,+\,\frac{2}{Z(\rho_0\,\pi\,b_0^2(Y)\,G(1))} \,=\, 1\,+\,2\,\left(1\,-\,\rho_0\,\pi\,b_0^2(Y)\,G(1)\right)\,. \tag{64}$$

Therefore, we obtain for $G(1)$:

$$G(1) \,=\, \frac{3}{1\,+\,\rho_0\,\pi\,b_0^2(Y)} \tag{65}$$

that in the high energy limit $Y \,\gg\, 1$ gives $\rho_0\,\pi\,b_0^2(Y) \,\ll\, 1$ and

$$G(1) \,\approx\, 3 \tag{66}$$

for any value of ρ_0. The functions $V(1)$ and $W(1)$ could be found as well and they have the following form:

$$V(1) \,=\, 1\,-\,\frac{1}{G(1)} \tag{67}$$

and

$$W(1) \,=\, 2\,\frac{G(1)\,-\,1}{G(1)^2}\,. \tag{68}$$

A numerical values of $\alpha \,=\, 0.8$ and $G(\infty) \,=\, 4.6$ are calculated in [8].

4.2 Parameters of the solution and fluid state of the gas

In order to calculate parameter A from the Eq. (46) we use the condition on the front of shock wave at initial time $t \,=\, 0$:

$$r(t\,=\,0) \,=\, A\,t_0^{\alpha} \,=\, R \tag{69}$$

with R from Eq. (3). So far we obtain

$$A \,=\, \frac{R}{t_0^{\alpha}}\,. \tag{70}$$

On another hand, the velocity of the shock wave at the initial moment of the time may be found from Eq. (48):

$$c_{sh} \,=\, V(1)\,\frac{\alpha\,R}{t_0}\,, \tag{71}$$

that gives

$$t_0 \,=\, \alpha\,V(1)\,\frac{R}{c_{sh}}\,. \tag{72}$$

Finally we obtain:

$$A \,=\, \frac{R}{t_0^{\alpha}} \,=\, \left(\frac{c_{sh}}{\alpha\,V(1)}^{\alpha}\right)\,R^{1\,-\,\alpha}\,. \tag{73}$$

Now we consider the pressure, Eq. (49), achieved in the system of interest with known solution for G, V, W functions:

$$P \,=\, \frac{\alpha^2\,r^2}{2\,(t\,-\,t_0)^2}\,\sigma_0\,G(\xi)\,W(\xi) \,=\, \frac{\alpha^2\,\sigma_0\,G(\xi)\,K_W}{2}\,A^{2/\alpha}\,r^{2\,-\,2/\alpha} \tag{74}$$

Before the compression of the gas the state equation of the gas is the usual one:

$$T_0 \,=\, \frac{P_0}{\rho_0}\,, \tag{75}$$

whereas at time of compression Eq. (20) holds:

$$T(\xi) \,=\, \frac{P(\xi)}{\rho(\xi)\,Z(\eta)}\,. \tag{76}$$

Therefore, with the pressure from Eq. (74) we obtain for the temperature ratio:

$$\frac{T(\xi)}{T_0} \,=\, \frac{\alpha^2\,\sigma_0\,\rho_0}{2\,\rho(\xi)\,P_0\,Z(\eta)}\,G(\xi)\,K_W\,A^{2/\alpha}\,r^{2\,-\,2/\alpha}\,. \tag{77}$$

Simplifying the expression we obtain finally

$$T(\xi) \,=\, \frac{m\,c_{sh}^2}{2\;Z(\eta)\,V(1)^2}\;K_W\,\left(\frac{R}{r}\right)^{2/\alpha\,-\,2}\,. \tag{78}$$

The maximum temperature in the center of area is achieved when $\xi\,\rightarrow\,\infty$ at $r\,\rightarrow\,0$. Taking this limit and changing r in the expression by the minimal possible distance Eq. (9), that smooth out the divergence, we obtain the maximum temperature achieved in the system in comparison to the initial temperature $T_0\,=\,P_0/\rho_0$:

$$T(\xi = \infty) \,=\, e^{\Delta\,Y}\;\frac{m\,c_{sh}^2}{2\;Z(\rho_0\,\pi\,b_0^2(Y)\,G(\infty)\,)\,V(1)^2}\;K_W\,\left(\frac{R}{\tilde{b}_0(Y)}\right)^{2/\alpha\,-\,2} \tag{79}$$

with

$$\Delta \,=\, \frac{1\,-\,\alpha}{\alpha}\,\approx\,0.25. \tag{80}$$

From another hand, we could compare the maximum achieved temperature with the temperature $T_0\,=\,T(\xi\,=\,1)$ on the edge of shock wave. In this case we have:

$$\frac{T(\xi)}{T(1)} \,=\, \frac{Z(\eta(1))}{G(1)\,Z(\eta(\xi))}\,\left(\frac{R}{r}\right)^{2/\alpha\,-\,2} \tag{81}$$

and again smoothing out the divergence we obtain:

$$\frac{T(\infty)}{T(1)} = e^{\Delta Y} \frac{Z(\rho_0 \, \pi \, b_0^2(Y) \, G(1))}{G(1) \, Z(\rho_0 \, \pi \, b_0^2(Y) \, G(\infty)\,)} \left(\frac{R}{\tilde{b}_0(Y)} \right)^{2/\alpha - 2} . \tag{82}$$

The resulting expressions Eq. (79), Eq. (82) is interesting from the following point of view. Taking ratio of temperatures in Eq. (82) from Eq. (19) as

$$\frac{T(\infty)}{T(1)} \propto \frac{T_{cr}}{T(Y=0)} \approx e^{5Y/2} \tag{83}$$

we arrive to the following expression:

$$\frac{R}{\tilde{b}_0(Y)} \approx e^{\Delta' \, Y} > 1 \tag{84}$$

with

$$\Delta' = \frac{7\alpha - 2}{4 - 4\alpha} \tag{85}$$

It is well known, see [11] for example, that when the characteristic parameter of the system, which is R in our case, is larger than the average distance between particle, which is $\tilde{b}_0(Y)$ in our case approximately, then we can consider a fluid flow instead initial gas state[4]. Thereby we obtain an important analytical result: at large enough external pressure, which is characterized by the rapidity in the present model, we can trace a dependence of the transition from gas phase to the liquid phase on the given external parameters. This is interesting especially because, as it wide accepted, the initial stages of quark-gluon plasma evolution is well described in the framework of hydrodynamical approach , for the review of the subject see [1] and [4] for the description of hydrodynamical expansion of the state.

5 Boltzmann equation for self-consistent field

In this section we consider a kinetic approach to our problem, which we reformulate in comparison to the Section 2 - Section 4. Namely, now we consider a gas of charged particles of the same kind interacting now in 3 dimensional space [5] which are under the pressure of external particles. When dissipative processes are negligible and local equilibrium is achieved, the Vlasov approach to the Boltzmann equation is valid. We assume that this is the case and, thereby, we consider the following Boltzmann equation for the one-particle distribution function in Vlasov's approximation of self-consistent fields:

$$\frac{\partial\, f(r,p,t)}{\partial\, t} + v \, \frac{\partial\, f(r,p,t)}{\partial\, r} + F(r) \, \frac{\partial\, f(r,p,t)}{\partial\, p} = 0 \tag{86}$$

[4]The ratio opposite to Eq. (84) ratio is an analog of Knudsen number in hydrodynamics.

[5]We formulate the problem in 3 dimensions because of importance of longitudinal dimension in real high energy interactions. The radial symmetry of the problem in this formulation is preserved as well and in the following we assume that all vectors of interest are radial. Therefore, the vector notations will be use only when it will be need in.

where we adopted the radial symmetry of the problem and where

$$F(r)\,=\,-\,\frac{\partial\,V_2(r)}{\partial\,r}\,+\,q\,E\,=\,-\,\frac{\partial\,V_2(r)}{\partial\,r}\,-\,q\,\frac{\partial\,\phi_0(r)}{\partial\,r} \tag{87}$$

is a force which consists of two terms arose from self-consistent electrical potential and external potential is similar to the one from Eq. (4):

$$V_2(r)\,=\,\frac{C}{2\,\pi\,R^2}\,e^{Y}\,f(\,r\,)\,. \tag{88}$$

Here $f(r)$ is some function of r and $E\,=\,-\,grad\,\phi_0(r)$ in Eq. (87) is a self-consistent electric field created by our charged particles. This field is assumed to be weak enough in order to justify a linear approximation for the distribution function. We assume also that the magnetic field is small and we neglect the magnetic filed in our calculations. The Vlasov's system of equations include Maxwell's equations for the electric field:

$$rot\,E\,=\,0\,,\quad div\,E\,=\,4\,\pi\,q\,n\,\int\,f(r,p,t)\,d^3\,p\,, \tag{89}$$

where n is a particle's density.

5.1 Linear approximation: equilibrium state

We solve the Boltzmann equation Eq. (86) by approximation of distribution function with some linear correction to the equilibrium state

$$f(r,p,t)\,=\,f_0(r,p)\,+\,f_1(r,p,t)\,,\;\;f_1\,\ll\,f_0 \tag{90}$$

and some linear correction to the constant electrical field

$$E\,=\,E^0\,+\,E^1\,. \tag{91}$$

We are looking for an equilibrium[6] state at the presence of external pressure in initial condition for the equation. Therefore, the Boltzmann equation which describes this equilibrium state has the following form:

$$v\,\frac{\partial\,f_0(r,p)}{\partial\,r}\,+\,F(r)\,\frac{\partial\,f_0(r,p)}{\partial\,p}\,=\,0\,, \tag{92}$$

whose solution is the Boltzmann-Maxwell distribution function:

$$f_0(r,p)\,=\,\frac{1}{(\,2\,\pi\,k_B\,T\,)^{3/2}}\,e^{-\,\frac{p^2}{2mk_BT}\,-\,\frac{V_2(r)}{k_BT}\,-\,\frac{q\,\phi_0(r)}{k_BT}}\,. \tag{93}$$

[6]In general, this state is quasi-equilibrium, the external pressure acts during finite period of time. Nevertheless, we assume, that this time period is long enough and we could consider this state as an equilibrium one during the early stages of system's evolution which we are interesting in.

The Maxwell equation for the electric field Eq. (89), therefore, reduces to the Poisson equation which has the following form in the limit of high temperatures (large kinetic energies of the particles in comparison to the potential energy):

$$- \Delta\phi_0(r) \,=\, 4\,\pi\,q\,n \left(1 \,-\, \frac{V_2(r)}{k_B T} \,-\, \frac{q\,\phi_0(r)}{k_B T}\right) \tag{94}$$

or

$$\Delta\,\phi_0(r) \,-\, 4\,\pi\,q^2\,n\,\frac{\phi_0(r)}{k_B T} \,=\, -\,4\,\pi\,q\,n \left(1 \,-\, \frac{V_2(r)}{k_B T}\right)\,. \tag{95}$$

We could rewrite it in the following form:

$$\Delta\,\phi_0(r) \,-\, \frac{\phi_0(r)}{r_D^2} \,=\, -\,Q_0 \,+\, \frac{V_2(r)}{q\,r_D^2}\,, \tag{96}$$

with $Q_0 \,=\, \frac{4\,\pi\,q\,n}{N}$ as the charge density and

$$r_D^2 \,=\, \frac{k_B\,T}{4\,\pi\,q^2\,n} \tag{97}$$

as the Debye length. Taking, as simplest example, the potential Eq. (1) in form of Eq. (4)

$$V_2(r) \,=\, \frac{C}{2\,\pi\,R^2}\,e^Y\,\theta\,(R^2 \,-\, r^2\,)\,, \tag{98}$$

we are looking for two solutions in two different regions:

1. Region where $r \,>\, R$. In this region the potential Eq. (1) is zero and we obtain the usual screening equation for the electric potential:

$$\Delta\,\phi_0(r) \,-\, \frac{\phi_0(r)}{r_D^2} \,+\, Q_0 \,=\, 0\,, \tag{99}$$

whose solution is

$$\phi_0(r) \,=\, Q_0\,r_D^2 \,+\, C_0\,\frac{e^{-\,r\,/\,r_D}}{r}\,. \tag{100}$$

2. Region where $r \,<\, R$. In this region we have

$$\Delta\,\phi_0(r) \,-\, \frac{\phi_0(r)}{r_D^2} \,+\, Q_0 \,-\, \frac{C}{2\,\pi\,R^2\,q\,r_D^2}\,e^Y \,=\, 0 \tag{101}$$

with solution for the potential

$$\phi_0(r) \,=\, Q_0\,r_D^2 \,+\, C_0'\,\frac{e^{-\,r\,/\,r_D}}{r} \,-\, \frac{C}{2\,\pi\,R^2\,q}\,e^Y\,. \tag{102}$$

Matching the solutions at $r \,=\, R$ we obtain finally:

1. In the region where $r \,>\, R$

$$\phi_0(r) \,=\, Q_0\,r_D^2 \,+\, C_0\,\frac{e^{-\,r\,/\,r_D}}{r}\,. \tag{103}$$

2. In the region where $r < R$

$$\phi_0(r) = Q_0 r_D^2 + C_0 \frac{e^{-r/r_D}}{r} + \frac{C e^Y}{2\pi R^2 q}\left(\frac{R}{r} e^{\frac{R-r}{r_D}} - 1\right) = Q_0^{'} r_D^2 + C_0 \frac{e^{-r/r_D}}{r}. \qquad (104)$$

with

$$Q_0^{'} = Q_0 + \frac{C e^Y}{2\pi R^2 r_D^2 q}\left(\frac{R}{r} e^{\frac{R-r}{r_D}} - 1\right) \qquad (105)$$

as new non-screened charge arose due the external pressure. The value of C_0 is determined by some additional boundary conditions which we do not consider in the problem.

The Debye length, Eq. (97), depends on the particles density n and we could define the plasma's parameter as

$$\mu = \frac{1}{n \int^{r_D(n)} d^3x \int d^3p\, f_0(r,p)} \ll 1. \qquad (106)$$

Requiring the validity of this parameter in both cases of presence and absence of external field V_2 we obtain, that in the case of the existing of external pressure the Debye radius is smaller than in the case of the absence of the pressure:

$$(r_D(Y))_{V_2 \neq 0} < (r_D)_{V_2 = 0}. \qquad (107)$$

It is known, see [11], that the hydrodynamic description of the process is possible when the characteristic external parameter of the system, in our case it is R from Eq. (3), is larger than the r_D:

$$R > r_D. \qquad (108)$$

Due to the external pressure the Debye radius decreases, and, therefore, we arrive to the inequality Eq. (108) at some large enough value of Y. We see, similarly to the previously obtained results, that we can trace the transition from the gas to the fluid state of the system by analytical methods where dependence on the external parameters in the conditions of the transition is introduced[7]. Therefore, perhaps, this approach can be useful in analytical research of the initial stages of evolution of quark-gluon plasma at high energy collisions.

Now, in the linear approximation over $f_0(r,p)$, we have the Boltzmann equation

$$\frac{\partial f_1(r,p,t)}{\partial t} + v \frac{\partial f_1(r,p,t)}{\partial r} + \left(-\frac{\partial V_2(r)}{\partial r} + q E^0\right) \frac{\partial f_1(r,p,t)}{\partial p} + q E^1 \frac{\partial f_0(r,p)}{\partial p} = 0 \qquad (109)$$

together with the Maxwell's equations for the electric field:

$$rot\, E^1 = 0, \quad div\, E^1 = 4\pi q n \int f_1(r,p,t)\, d^3 p, \qquad (110)$$

where

$$E^0 = -grad\, \phi_0(r) \qquad (111)$$

with $\phi_0(r)$ given by Eq. (103) and Eq. (104). Solutions for the $f_1(r,p,t)$ distribution function and E^1 field we are deriving in the Appendix B of the manuscript.

[7]In general, instead rapidity dependence, any other parameters, which characterizes the external pressure and reduce the problem to equilibrium one, can be introduced.

6 Conclusion

In our paper we considered a toy model of particles interacting at high energy based on the effect of inertial confinement like in plasma physics. There are few main problems which were considered in the framework of the model and which can be interesting for the case of quark-gluon plasma description as well. These problems are the following: the transition from the gas state to the liquid state of the system and a change of the transport properties of the system of interacting particles during this transition.

The trace of the transition to the liquid phase can be characterized by the so called Knudsen number in hydrodynamics, see [11]. In our model we demonstrate how much during the evolution of the system this number is changing and starting to be large[8] , see Eq. (84) and Eq. (108). The largeness of this ratio is obtained in two different calculation schemes: by the description of the process in the shock waves framework and in the framework of Vlasov's equation. Both approaches are based on the assumption that in almost ideal liquid, which is QGP (see [1]), a contribution of dissipative processes is small, i.e. dissipation could be manifested on the scales much smaller then the scales considered in the paper. In this case the compression of the "internal" gas area by external pressure could be considered as an adiabatic process for the convergent to the center of the area of the shock wave. This problem of searching for self-similar solutions for the case of non-polytropic gas was considered in [8], we adopted the results of this paper for our aims. Also, the assumption on the absence of the large-scale dissipative processes allows us to consider the Vlasov's approximation of the Boltzmann equation for the self-consistent field for our system as an acceptable as well. The external pressure, introduced in the mode in correspondence to the receipts of [3], reduces the highly non-equilibrium process to an equilibrium one in some external field. It is interesting to understand a possibility of similar description of the collision process in the framework of QCD, this work we are planning for the future.

As mentioned above, the large value of the ratio Eq. (84) and Eq. (108) is a sign of the liquid phase creation. The dependence of this ratio on the external parameters of the problem is an important result, because the main tool in quark-gluon plasma evolution is the hydrodynamical description of the plasma state, see [1]. Therefore, our approach in application to QGP can serve as a description of the initial states of the hydrodynamical evolution. Namely, it will be interesting task to use the proposed approach for the definition of the initial states of hydrodynamical expansion in the framework of Landau model, see [4] and for calculation of multiplicities of the produced partons in high energy interactions, see [12] and references therein.

The QCD QGP is an almost ideal fluid, where interactions with large coupling constant between the participants of collision are revealed, see [1] for review. This fluid has very special transport

[8]Our ratio is opposite to the usual Knudsen number, therefore in our approach the liquid state is characterized by the large value of the ratio.

properties, which so far can be partially explained only in framework of AdS/CFT duality model; only there the calculations in the strong coupling constant domain are possible. Nevertheless, in our approach we tried to consider the subject of the transport properties of the system under the external pressure using well known Enskog calculation scheme, see Eq. (20) and [7]. It is important to underline also, that in Enskog approach we calculate the corrections to the transport coefficients caused by the external influence on the gas whereas the initial values of the parameters are given. Our results demonstrate that in this case the transport properties of the system of interacting particles near the transition into the liquid phase, see Eq. (34)-Eq. (36), are similar in some sense to the properties of the quark-gluon plasma obtained in [9]. Namely, we obtain similar rise of bulk viscosity when temperature is rising. The explanation of this effect in our model is simple. In Enskog approximation the bulk viscosity is proportional to the density of the state. The density is growing toward some constant value whereas the system of interest undergo a gas-liquid transition. When the state of ideal liquid is created finally then the density is constant due the incompressibility of the state and, therefore, the bulk viscosity as well. Another interesting result is an equation Eq. (39); we see that this ratio is small in our framework similarly to what is observed in QGP , see for review and explanations [1].

Finally we conclude, that the main purpose of the exploration of our toy model was an understanding of the main principles and methods of the approach for it's generalization at the case of quark-gluon plasma. There are the following discussed problems which we found also interesting to investigate at the case of the quark-gluon plasma:

a The dynamical change of the Debye length of the plasma under the external pressure. This change indicates the gas-liquid transition by the change of the Knudsen number of the system. In our paper the Debye length change dynamics was depend on the model introduced potential for the external field pressure. Nevertheless, if we have a plasma state under some external pressure the similar effect must be observed for any kind of the plasmas.

b Calculations of the one-particle distribution function and self-consistent fields in the Vlasov's approximation for the dense compressed drop of the partons in the field of external particles. This question is especially interesting because this function can clarify the transport properties of the system which are very unusual at the case of QGP.

The obtained results show that these tasks can be achieved in the approach correctly reformulated for the case of QCD QGP that we plan as our future work.

Appendix A: gas of interacting particles

In this appendix we shortly remind the main facts concerning a virial expansion, see the detailed derivation in [13]. We consider a gas of N interacting particles each with mass m on a plane as a gas of hard disk with small thickness and radius r_0. The energy of the gas in the classical limit is given by the well known expression:

$$E(p,q) = \sum_{i=1}^{N} \frac{p_i^2}{2\,m} \,+\, U(b_1,...,b_N) \tag{A.1}$$

where as usual the first term is the kinetic energy of N particles, U is a potential energy of their mutual interactions and $b_1,...,b_N$ their coordinates. The grand partition function for this Hamiltonian is

$$Q(\mu,T,S) \,=\, e^{-\beta\,\Omega} \,=\, \sum_{N=0}^{\infty} e^{\beta\,\mu}\,\frac{1}{N!}\int \prod_{i=1}^{N} \frac{d^2\vec{p}_i\,d^2\vec{b}_i}{h^2}\,\exp(-\beta\sum_{i=1}^{N}\frac{p_i^2}{2\,m}\,)\,\,\exp(-\beta\;U(b_1,...,b_N))\,. \tag{A.2}$$

where Ω is the grand potential of the problem. As usual, integrating over momenta, we obtain

$$Q(\mu,T,S) \,=\, e^{-\beta\,\Omega} \,=\, \sum_{N=0}^{\infty}\left(\frac{e^{\beta\,\mu}}{\lambda^2}\right)^N \frac{Z_N(S,\beta)}{N!} \tag{A.3}$$

and

$$Z_N(S,\beta) \,=\, \int ... \int d^2\vec{b}_1,...,d^2\vec{b}_N\,\exp\left(-\beta\;U(b_1,...,b_N)\right)\,. \tag{A.4}$$

Here λ is the De Broglie length of the quark corresponding to the average energy β:

$$\lambda \,=\, \left(\frac{h^2\,\beta}{2\,\pi\,m}\right)^{1/2}\,. \tag{A.5}$$

The potential of the problem is given by

$$\Omega \,=\, -\frac{1}{\beta}\,\ln\left(1+\frac{e^{\beta\,\mu}}{\lambda}\,S\,+\,\frac{e^{2\,\beta\,\mu}}{2!\,\lambda^2}\int\int d^2\vec{b}_1\,d^2\vec{b}_2\,\exp\left(-\beta\;U(b_1,b_2)\,\right)\,+...\right)\,. \tag{A.6}$$

Here, we used

$$\int d^2\vec{b} \,=\, S \,=\pi\,R^2\,, \tag{A.7}$$

where R^2 is the characteristic radius of the problem. In the following we will define and consider only pairwise interaction between the particles, namely we have

$$U(b_1,b_2) \,=\, U(|b_1-b_2|) \,=\, U(b_{12}) \,=\, U(b) \,=\, U_{12} \tag{A.8}$$

and therefore, in the relative coordinates of the center mass we reduce the multiplicity of the integrated functions and obtain an additional S factor in the integrals:

$$\Omega \,=\, -P\,S \,=-\frac{1}{\beta}\,\ln\left(1\,+\,S\,\frac{e^{\beta\,\mu}}{\lambda^2}\,+\,S\,\frac{e^{2\,\beta\,\mu}}{2!\,\lambda^4}\int\int d^2\vec{b}_{12}\,\exp\left(-\beta\;U_{12}\,\right)\,+...\right)\,. \tag{A.9}$$

Introducing variable ζ

$$\zeta \,=\, \frac{e^{\beta\,\mu}}{\lambda^2} \tag{A.10}$$

we obtain the expression for the potential in the form of the series in ζ

$$\Omega \,=\, -P\,S \,=\, -\frac{S}{\beta}\sum_{n=1}^{\infty}\frac{J_n}{n!}\,\zeta^n\,. \tag{A.11}$$

We will take into account only two first terms of this series with the following J_1 and J_2:

$$J_1 \,=\, 1\,, \quad J_2 \,=\, \int\!\!\int d^2\vec{b}_{12}\;(\,\exp\,(-\beta\;U_{12}\,)\,-\,1\,)\,. \tag{A.12}$$

The number of particles in this gas we obtain as usual

$$N = -\left(\frac{\partial\,\Omega}{\partial\,\mu}\right)_{T,S} \tag{A.13}$$

and because $\partial\,\zeta\,/\,\partial\,\mu \,=\, \beta\,\zeta$ we finally have:

$$N \,=\, S\sum_{n=1}^{\infty}\frac{J_n}{(n-1)!}\,\zeta^n\,. \tag{A.14}$$

Excluding from Eq. (A.11) and Eq. (A.14) the variable ζ, we obtain in the second order approximation the equation of state for our gas:

$$P \,=\, \frac{NT}{S} \,-\, \frac{N^2\,T}{2\,S^2}\,J_2\,, \tag{A.15}$$

where $T = 1/\beta$.

Appendix B: deviation from equilibrium state and non static electrical field of the gas in Vlasov's approximation

Proceeding with the equation Eq. (90) we perform a following substitution:

$$f_1(r,p,t) \,=\, \bar{f}_1(r,p,t)\, f_0(r,p) \tag{B.1}$$

with $f_0(r,p)$ from Eq. (93). Rewriting Eq. (109) we obtain:

$$\frac{\partial\, \bar{f}_1(r,p,t)}{\partial\, t} \,+\, \frac{p}{m}\, \frac{\partial\, \bar{f}_1(r,p,t)}{\partial\, r} \,+\, F\, \frac{\partial\, \bar{f}_1(r,p,t)}{\partial\, p} \,-\, \frac{q\, E^1}{m\, k_B\, T}\, p \,=\, 0\,, \tag{B.2}$$

where F is given by Eq. (87). We consider a linear approximation over the equilibrium distribution, therefore, we could write the external force in the equation in following form

$$F(r) \,\rightarrow\, \bar{F} \,=\, F(\bar{r}) \,=\, -\, \left(\frac{\partial\, V_2(r)}{\partial\, r} \right)_{r\,=\,\bar{r}} \,+\, q\, E^0(\bar{r})\,. \tag{B.3}$$

with r

$$\bar{r} \,=\, \frac{\int\, r\, f_0(r,p)\, d^3x\, d^3p}{\int\, f_0(r,p)\, d^3x\, d^3p}\,. \tag{B.4}$$

Thereby we have the following equation for the $\bar{f}_1(r,p,t)$ distribution function:

$$\frac{\partial\, \bar{f}_1(r,p,t)}{\partial\, t} \,+\, \frac{p}{m}\, \frac{\partial\, \bar{f}_1(r,p,t)}{\partial\, r} \,+\, \bar{F}\, \frac{\partial\, \bar{f}_1(r,p,t)}{\partial\, p} \,-\, \frac{q\, E^1}{m\, k_B\, T}\, p \,=\, 0 \tag{B.5}$$

Performing Fourier transform of $\bar{f}_1(r,p,t)$ and $E^1(r,t)$

$$\bar{f}_1(r,p,t) \,=\, \int \frac{d\omega}{2\,\pi} \int \frac{d^3k}{(2\,\pi)^3}\, e^{-i\omega\, t\, +\, i\, r\, k}\, \phi_0(k,p,\omega) \tag{B.6}$$

$$E^1(r,t) \,=\, \int \frac{d\omega}{2\,\pi} \int \frac{d^3k}{(2\,\pi)^3}\, e^{-i\omega\, t\, +\, i\, r\, k}\, \tilde{E}^1(k,\omega) \tag{B.7}$$

we obtain finally a nonlinear differential equation of the first order over momenta:

$$\frac{\partial\, \phi_0}{\partial\, p} \,+\, a\, p\, \phi_0 \,-\, b\, \phi_0 \,-\, c\, p \,=\, 0\,. \tag{B.8}$$

Here

$$a \,=\, \frac{ik}{m\, \bar{F}}\,, \;\; b \,=\, \frac{i\, \omega}{\bar{F}}\,, \;\; c \,=\, \frac{q\, \tilde{E}^1}{m\, k_B\, T\, \bar{F}}\,. \tag{B.9}$$

The solution of this equation, with additional condition that at $p \,\rightarrow\, 0\, (T \,\rightarrow\, 0\,)$ the solution is static, i.e. $\phi_0(k, p \,=\, 0\,, \omega) \,=\, 0\,$, is the following function:

$$\phi_0(k,p,\omega) \,=\, c\, e^{-a\, p^2\, /\, 2\, +\, b\, p} \int_0^p p^{'}\, e^{a\, p^{'2}\, /\, 2\, -\, b\, p^{'}}\, dp^{'}\,. \tag{B.10}$$

Thereby we obtain for the correction to the equilibrium state:

$$\phi_0(k,p,\omega) \,=\, \frac{q\, \tilde{E}^1}{m\, k_B\, T\, \bar{F}}\, e^{-a\, p^2\, /\, 2\, +\, b\, p} \int_0^p p^{'}\, e^{a\, p^{'2}\, /\, 2\, -\, b\, p^{'}}\, dp^{'}\,. \tag{B.11}$$

We see, that this correction is suppressed in comparison to the distribution function Eq. (93) by the factor $\frac{q\tilde{E}^1}{\bar{F}}$. Substituting this expression back in Eq. (B.6), we have:

$$\bar{f}_1(r,p,t) \;=\; \frac{q}{m\,k_B\,T\,\bar{F}}\int_0^p dp^{'}\,p^{'}\int\frac{d\omega}{2\,\pi}\int\frac{d^3k}{(2\,\pi)^3}\,\tilde{E}^1(k,\omega)\,e^{-i\omega\left(t+\frac{p^{'}}{\bar{F}}-\frac{p}{\bar{F}}\right)+i\,k\left(r+\frac{p^{'2}}{2\,m\,\bar{F}}-\frac{p^2}{2\,m\,\bar{F}}\right)}. \tag{B.12}$$

Performing Fourier transform again, we obtain for our function in (r,t) representation:

$$\bar{f}_1(r,p,t) \;=\; \frac{q}{m\,k_B\,T\,\bar{F}}\int_0^p dp^{'}\,p^{'}\,E^1(r\,+\,\frac{p^{'2}\,-\,p^2}{2\,m\,\bar{F}}\,,t\,+\,\frac{p^{'}\,-\,p}{\bar{F}}\,)\,. \tag{B.13}$$

This correction to the static distribution function determines also the non-static electric field which we consider further.

Using Eq. (89), we find the equation for the correction E^1 to the electric field:

$$div\,E^1(r,t) \;=\; \frac{4\,\pi\,q^2\,n}{m\,k_B\,T\,\bar{F}}\int d^3\,p\int_0^p dp^{'}\,p^{'}\,E^1(r\,+\,\frac{p^{'2}\,-\,p^2}{2\,m\,\bar{F}}\,,t\,+\,\frac{p^{'}\,-\,p}{\bar{F}}\,)\,f_0(r,p) \tag{B.14}$$

with with $f_0(r,p)$ from Eq. (93), see also Eq. (B.1). This equation is highly non-linear and non-local and, perhaps, a precise solution of the equation can be found only by numerical methods. In order to investigate approximate solutions of the equation, we perform Fourier transform of the functions in Eq. (B.14):

$$k\tilde{E}^1(k,\omega) = \frac{4\pi\,q^2\,n}{m\,k_B\,T\bar{F}}\int d^3p\int_0^p dp^{'}p^{'}\int\frac{d^3k^{'}}{(2\pi)^3}\tilde{E}^1(k-k^{'},\omega)e^{-i\omega\left(\frac{p^{'}-p}{\bar{F}}\right)+i(k-k^{'})\left(\frac{p^{'2}-p^2}{2m\bar{F}}\right)}\tilde{f}_0(k^{'},p)\,, \tag{B.15}$$

where

$$f_0(r,p) \;=\; \int\frac{d^3k}{(2\,\pi)^3}\,e^{i\,r\,k}\,\tilde{f}_0(k,p)\,. \tag{B.16}$$

In the right hand side of the equation the oscillating integral over $k^{'}$ is not vanishing when

$$k^{'}\,\propto\,\frac{2m\bar{F}}{p^{'2}-p^2}\,\propto\,\frac{\bar{F}}{k_B\,T}\,\ll\,1 \tag{B.17}$$

in our approximation of the weak external field. Therefore, in the region where

$$k\,>\,k^{'} \tag{B.18}$$

in the first approximation over $k^{'}$ we have:

$$k\tilde{E}^1(k,\omega) \approx \tilde{E}^1(k,\omega)\frac{4\pi\,q^2\,n}{m\,k_B\,T\bar{F}}\int d^3p\int_0^p dp^{'}p^{'}\int\frac{d^3k^{'}}{(2\pi)^3}e^{-i\omega\left(\frac{p^{'}-p}{\bar{F}}\right)+i(k-k^{'})\left(\frac{p^{'2}-p^2}{2m\bar{F}}\right)}\tilde{f}_0(k^{'},p) -$$

$$-\;\frac{\partial\,\tilde{E}^1(k,\omega)}{\partial\,k}\,\frac{4\pi\,q^2\,n}{m\,k_B\,T\bar{F}}\int d^3p\int_0^p dp^{'}p^{'}\int\frac{d^3k^{'}}{(2\pi)^3}\,k^{'}\,e^{-i\omega\left(\frac{p^{'}-p}{\bar{F}}\right)+i(k-k^{'})\left(\frac{p^{'2}-p^2}{2m\bar{F}}\right)}\tilde{f}_0(k^{'},p) \tag{B.19}$$

or, simplifying the notations we obtain:

$$\frac{\partial \tilde{E}^1(k,\omega)}{\partial k} \, - \, \tilde{E}^1(k,\omega) \, (\, C_1(\omega,k) \, - \, C_2(\omega,k)\, k \,) \, = \, 0 \, , \tag{B.20}$$

where

$$C_1(\omega,k) \, = \, \frac{\int d^3p \int_0^p dp^{'} p^{'} \, e^{-i\omega\left(\frac{p^{'}-p}{\bar{F}}\right)+i\,k\left(\frac{p^{'2}-p^2}{2m\bar{F}}\right)} \, f_0(\frac{p^2-p^{'2}}{2m\bar{F}}, p)}{\int d^3p \int_0^p dp' p' \int \frac{d^3k'}{(2\pi)^3}\, k' \, e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)+i\,(k-k')\left(\frac{p'^2-p^2}{2m\bar{F}}\right)} \, \tilde{f}_0(k', p)} \tag{B.21}$$

and

$$C_2(\omega, \, k) \, = \, \frac{m\, k_B \, T\bar{F}}{4\pi \, q^2 \, n \, \int d^3p \int_0^p dp' p' \int \frac{d^3k'}{(2\pi)^3}\, k' \, e^{-i\omega\left(\frac{p'-p}{\bar{F}}\right)+i\,(k-k')\left(\frac{p'^2-p^2}{2m\bar{F}}\right)} \, \tilde{f}_0(k', p)} \, . \tag{B.22}$$

An integration of Eq. (B.19) gives

$$\tilde{E}^1(k,\omega) \, = \, C_0(\omega) \, e^{\int_0^k \, C_1(\omega,t)\, dt - \int_0^k \, C_2(\omega,t)\, t\, dt} \, . \tag{B.23}$$

We see, that the fluctuation of electric field in this region of k is suppressed by the large factor C_2 in the power of exponent.

In the opposite limit, when

$$k \, \sim \, k^{'} \, \propto \, \frac{2m\bar{F}}{p^{'2} - p^2} \, \propto \, \frac{\bar{F}}{k_B\, T} \, \ll \, 1 \tag{B.24}$$

we introduce new variable ϵ:

$$\epsilon \, = \, k \, - \, k^{'} .$$

Equation Eq. (B.19) will acquire the following form:

$$k\tilde{E}^1(k,\omega) = \frac{4\pi\, q^2\, n}{m\, k_B\, T\bar{F}} \int d^3p \int_0^p dp^{'} p^{'} \int \frac{d^3\epsilon}{(2\pi)^3} \tilde{E}^1(\epsilon,\omega) e^{-i\omega\left(\frac{p^{'}-p}{\bar{F}}\right)+i\epsilon\left(\frac{p^{'2}-p^2}{2m\bar{F}}\right)} \, \tilde{f}_0(k-\epsilon, p) \, . \tag{B.25}$$

Fourier transform of function Eq. (93) is

$$\tilde{f}_0(k,p) = \tilde{f}_0(k) f_0(p) = f_0(p) \int \, d^3r \, e^{-ir\,k} e^{-\frac{V_2(r)}{k_B T} - \frac{q\phi_0(r)}{k_B T}} \approx f_0(p) \left((2\pi)^3 \delta^3(k) - \frac{\tilde{V}_2(k)}{k_B T} - \frac{q\tilde{\phi}_0(k)}{k_B T} \right) \tag{B.26}$$

with

$$f_0(p) \, = \, \frac{1}{(\, 2\, \pi \, k_B \, T\,)^{3/2}} \, e^{-\frac{p^2}{2mk_B T}} \, , \tag{B.27}$$

therefore, in the first approximation of expansion over ϵ we obtain:

$$\tilde{E}^1(k,\omega) = \, \tilde{E}^1(k,\omega) \, \frac{4\pi \, q^2\, n}{m\, k_B \, T\bar{F}\, k} \int d^3p \, f_0(p) \int_0^p dp^{'} p^{'} \, e^{-i\omega\left(\frac{p^{'}-p}{\bar{F}}\right)+i\,k\left(\frac{p^{'2}-p^2}{2m\bar{F}}\right)} \, -$$

$$- \tilde{E}^1(0,\omega)\, \frac{4\pi\, q^2\, n\, \tilde{V}_2(k)}{m\,(k_B\,T)^2\, \bar{F}\, k} \int d^3p\, f_0(p) \int_0^p dp^{'} p^{'}\, e^{-i\omega\left(\frac{p^{'}-p}{\bar{F}}\right)} \int^{\frac{\bar{F}}{k_B\,T}} \frac{d\epsilon\, \epsilon^2}{2\pi^2} -$$

$$- \;\; \tilde{E}^1(0,\omega)\, \frac{4\pi\, q^3\, n\, \tilde{\phi}_0(k)}{m\,(k_B\,T)^2\, \bar{F}\, k} \int d^3p\, f_0(p) \int_0^p dp^{'} p^{'}\, e^{-i\omega\left(\frac{p^{'}-p}{\bar{F}}\right)} \int^{\frac{\bar{F}}{k_B\,T}} \frac{d\epsilon\, \epsilon^2}{2\pi^2}\,. \qquad \text{(B.28)}$$

Finally we obtain for our electric field:

$$\tilde{E}^1(k,\omega)\, \varepsilon \;=\; - \tilde{E}^1(0,\omega)\, \frac{2\, q^2\, n\, \bar{F}^2}{3\,\pi\, m\,(k_B\,T)^4\; k} \left(\frac{\tilde{V}_2(k)}{k_BT} + \frac{q\tilde{\phi}_0(k)}{k_BT} \right) \int d^3p\, f_0(p) \int_0^p dp^{'} p^{'}\, e^{-i\omega\left(\frac{p^{'}-p}{\bar{F}}\right)} \qquad \text{(B.29)}$$

where

$$\varepsilon \;=\; 1 \;-\; \frac{4\pi\, q^2\, n}{m\, k_B\, T \bar{F}\, k} \int d^3p\, f_0(p) \int_0^p dp^{'} p^{'}\, e^{-i\omega\left(\frac{p^{'}-p}{\bar{F}}\right) + i\,k\left(\frac{p^{'2}-p^2}{2m\bar{F}}\right)} \qquad \text{(B.30)}$$

is the dielectric constant of the problem.

References

[1] E.Shuryak, Nucl. Phys. B **195** 111 (2009).

[2] R. Kirschner, L. N. Lipatov and L. Szymanowski, Nucl. Phys. B **425**, 579 (1994); M. A. Braun, L. N. Lipatov, M. Y. .Salykin and M. I. Vyazovsky, Eur. Phys. J. C **71**, 1639 (2011).

[3] M.A.Leontovich, ZhETF **8**, 7, 844 (1938); Yu.L.Klimontovich, Sov. Phys. Usp. 26, 366 (1983).

[4] L.D.Landau, Izv. Akad. Nauk: Ser.Fiz.**17**, 51,(1953).

[5] S.Pfalzner, "An introduction to inertial confinement fusion", Taylor and Francis press.

[6] L. N. Lipatov, Sov. J. Nucl. Phys. **23**, 338 (1976) [Yad. Fiz. **23** (1976) 642]; E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **45**, 199 (1977) [Zh. Eksp. Teor. Fiz. **72**, 377 (1977)]; I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978) [Yad. Fiz. **28**, 1597 (1978)].

[7] S.Chapman and T.G.Cowling, "The mathematical theory of non-uniform gases", Cambridge University Press; D.M.Gass, J. Chem. Phys. **54**,1898 (1971).

[8] P.Gaspard and J.Lutsko, Phys. Rev. E **70**, 026306 (2004).

[9] D. Kharzeev and K. Tuchin, JHEP **0809**, 093 (2008).

[10] E. Levin, Heavy Ion Phys. **8**, 265 (1998).

[11] Yu.L.Klimontovich, Sov. Phys. Usp. 167, 23 (1997).

[12] D. Kharzeev, E. Levin and K. Tuchin, Phys. Rev. C **75**, 044903 (2007); E. K. G. Sarkisyan and A. S. Sakharov, Eur. Phys. J. C **70**, 533 (2010).

[13] L.D.Landau, E.M.Lifshitz and L.P.Pitaevskii, "Statistical Physics", Part 1 (Course Theoretical Physics, Volume 5).